Plate punching analysis

Code: Eurocode

Beam 8, Node 24

Input data

Materials

Concrete: **C16/20** (f_{ck} = 16 MPa)

Rebar steel: **B500A** (f_{ywk} = 500 MPa)

Geometry

Cross-section: **PCA_40x40**

Plate thickness: h = 20,0 cm

Rebar position:

a_x = 3,4 cm

a_y = 5,0 cm

Effective slab depth:

$d_x = h - a_x = 20{,}0 - 3{,}4 = 16{,}6$ cm

$d_y = h - a_y = 20{,}0 - 5{,}0 = 15{,}0$ cm

Mean effective slab depth:

$$d = \frac{d_x + d_y}{2} = \frac{16{,}6 + 15{,}0}{2} = 15{,}8 \text{ cm} = 0{,}158 \text{ m}$$

Result summary

u_0 = 1,6 m

u_1 = 3,3027 m

β = 1,350

$$\frac{v_{Ed}}{v_{Rd,c}} = \frac{689{,}28}{494{,}55} = 1{,}3938 > 1$$!!

$$\frac{v_{Ed,0}}{v_{Rd,max}} = \frac{1422{,}8}{2396{,}2} = 0{,}59379 < 1$$ ✔

A_{sw} = 2,87 cm²

$$\frac{v_{Ed}}{v_{Rd,cs}} = \frac{689{,}28}{689{,}29} = 0{,}99999 < 1$$ ✔

n_{sr} = 4

Internal forces

Load case: **Critical Min,Max. ([1,35*G] {1,5*0,7*Q1} (1,5*0,7*Q2+1,5*0,7*Q3+1,5*0,7*Q4+1,5*0,7*Q5+1,5*0,6*Wind+1,5*0,7*Q6))**

V_{Ed} = 266,43 kN (↑)

M_{Edy} = −0,79 kNm

M_{Edz} = 11,806 kNm

n_{Edx} = 1,1665 kN/m

n_{Edy} = 0,6804 kN/m

$$\sigma_{cp} = \frac{\sigma_{cx} + \sigma_{cy}}{2} = \frac{(-5{,}8325) + (-3{,}402)}{2} = -4{,}6173 \text{ KPa} = -0{,}0046173 \text{ MPa}$$

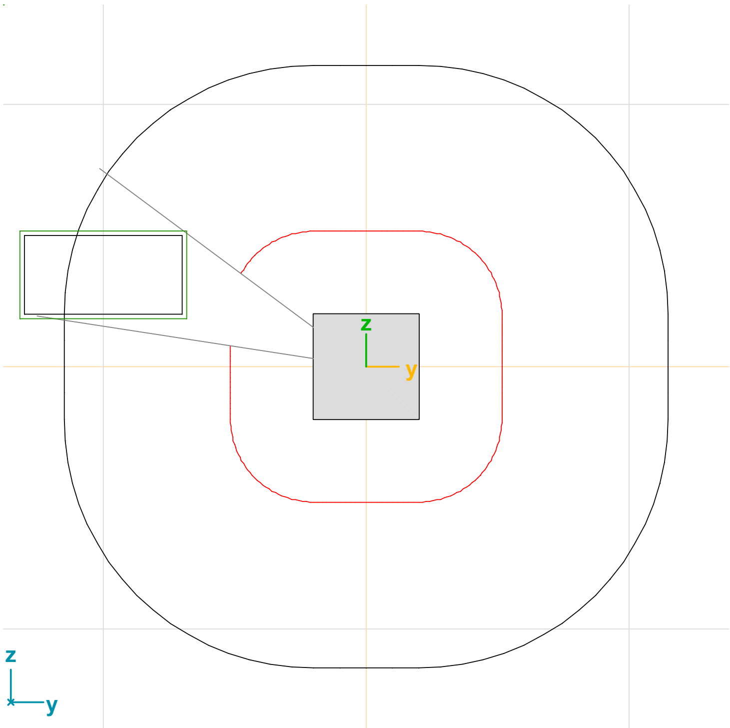
Length of the perimeter of the load area:

$u_0 = 2 \cdot (c_x + c_y) = 2 \cdot (0{,}4 + 0{,}4) = 1{,}6$ m

Basic control perimeter

The distance between the perimeter of the load area and the basic control perimeter: $2 \cdot d = 2 \cdot 0{,}158 = 0{,}316$ m

The length of the basic control perimeter: u_1 = 3,3027 m



Coefficient for consideration of load eccentricities in the load application

User-defined value

$\beta = 1{,}350$

Partial factors for materials

Design situation: Persistent and transient

Partial factor for concrete: $\gamma_c = 1{,}500$

Partial factor for reinforcing steel: $\gamma_s = 1{,}150$

$$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} = 1 \cdot \frac{16}{1{,}500} = 10{,}667 \text{ MPa} = 10667 \text{ KPa}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{500}{1{,}150} = 434{,}78 \text{ MPa} = 434783 \text{ KPa}$$

$$f_{ywd} = \frac{f_{ywk}}{\gamma_s} = \frac{500}{1{,}150} = 434{,}78 \text{ MPa} = 434783 \text{ KPa}$$

Checking the maximum punching shear resistance along the column perimeter

Maximum shear stress at the column perimeter: EN-1992-1-1 6.4.5 (3) (6.53)

$$v_{Ed,0} = \beta \cdot \frac{V_{Ed}}{u_0 \cdot d} = 1{,}350 \cdot \frac{266{,}43}{1{,}6 \cdot 0{,}158} = 1422{,}8 \text{ KPa}$$

Strength reduction factor for concrete cracked in shear:

$$v = 0{,}6 \cdot \left(1 - \frac{f_{ck}}{250}\right) = 0{,}6 \cdot \left(1 - \frac{16}{250}\right) = 0{,}562$$ EN-1992-1-1 6.2.2 (6) (6.6N)

The design value of the maximum punching shear resistance along the column perimeter:

$$v_{Rd,max} = 0{,}4 \cdot v \cdot f_{cd} = 0{,}4 \cdot 0{,}562 \cdot 10667 = 2396{,}2 \text{ KPa}$$ EN-1992-1-1 6.4.5.(3)

$$\frac{v_{Ed,0}}{v_{Rd,max}} = \frac{1422{,}8}{2396{,}2} = 0{,}59379 < 1 \text{ passed}$$

Checking the slab along the basic control perimeter

Maximum shear stress at the u_1 perimeter: EN-1992-1-1 6.4.3 (3) (6.38.)

$$v_{Ed} = \beta \cdot \frac{V_{Ed}}{u_1 \cdot d} = 1{,}350 \cdot \frac{266{,}43}{3{,}3027 \cdot 0{,}158} = 689{,}28 \text{ KPa}$$

Normal stresses in concrete (positive if compression):

$$\sigma_{cx} = -\frac{n_{Edx}}{h} = -\frac{1{,}1665}{20{,}0} = -5{,}8325 \text{ KPa}$$

$$\sigma_{cy} = -\frac{n_{Edy}}{h} = -\frac{0{,}6804}{20{,}0} = -3{,}402 \text{ KPa}$$

$$\sigma_{cp} = -0{,}0046173 \text{ MPa}$$

Bonded tension steel ratios in x and y direction:

$$\rho_{lx} = \frac{A_{slx}}{b_w \cdot d_x} = \frac{0{,}00075223}{1 \cdot 16{,}6} = 0{,}0045315$$

$$\rho_{ly} = \frac{A_{sly}}{b_w \cdot d_y} = \frac{0{,}00099548}{1 \cdot 15{,}0} = 0{,}0066365$$

$$\rho_l = \sqrt{\rho_{lx} \cdot \rho_{ly}} \le 0{,}02 = \sqrt{0{,}0045315 \cdot 0{,}0066365} \le 0{,}02 = 0{,}0054839$$

$$C_{Rd,c} = \frac{0{,}18}{\gamma_c} = \frac{0{,}18}{1{,}500} = 0{,}12$$ EN-1992-1-1 6.4.4. (1)

$$k = \min\left(1 + \sqrt{\frac{200}{d}}\ ;\ 2\right) = \min\left(1 + \sqrt{\frac{200}{158}}\ ;\ 2\right) = 2{,}000$$

where d is measured in mm

$$v_{min} = 0{,}035 \cdot k^{3/2} \cdot f_{ck}^{1/2} = 0{,}035 \cdot 2{,}000^{3/2} \cdot 16^{1/2} = 0{,}39598 \text{ MPa} = 395{,}98 \text{ KPa}$$ EN-1992-1-1 6.2.2 (1) (6.3N)

The design value of punching shear resistance of the slab without punching shear reinforcement: EN-1992-1-1 6.4.4 (1) (6.47)

$$v_{Rd,c} = \text{Max}\left(C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp} = 0{,}12 \cdot 2{,}000 \cdot (100 \cdot 0{,}0054839 \cdot 16)^{1/3} + 0{,}1 \cdot (-0{,}0046173) = \right.$$
$$\left. = 0{,}49455\ ;\ v_{min} + k_1 \cdot \sigma_{cp} = 0{,}39598 + 0{,}1 \cdot (-0{,}0046173) = 0{,}39552 \right) = 0{,}49455 \text{ MPa} = 494{,}55 \text{ KPa}$$

where f_{ck} is measured in N/mm^2

Shear utilization of the slab:

$$\frac{v_{Ed}}{v_{Rd,c}} = \frac{689{,}28}{494{,}55} = 1{,}3938 > 1$$ ‼ **Shear reinforcement is required.**

Design of punching shear reinforcement

Radial spacing of first perimeter of shear reinforcement from the convex perimeter of the column: $s_0 = 4{,}7$ cm

Radial spacing of perimeters of shear reinforcement: s_r = 11,9 cm = 0,1185 m

The angle between the shear reinforcement and the plane of the slab: α = 90,00 °

Effective design yield strength of the punching shear reinforcement: EN-1992-1-1 6.4.5 (1)

$$f_{ywd,eff} = 250 + 0{,}25 \cdot d = 250 + 0{,}25 \cdot 158 = 289{,}5 \text{ MPa} = 289500 \text{ KPa} \le f_{ywd} = 434783 \text{ KPa}$$

The area of shear reinforcement at the basic control perimeter:

$$A_{sw,c} = \frac{v_{Ed} - 0{,}75 \cdot v_{Rd,c}}{1{,}5 \cdot \frac{d}{s_r} \cdot f_{ywd,eff} \cdot \frac{1}{u_1 \cdot d} \cdot \sin\alpha} = \frac{689{,}28 - 0{,}75 \cdot 494{,}55}{1{,}5 \cdot \frac{0{,}158}{0{,}1185} \cdot 289500 \cdot \frac{1}{3{,}3027 \cdot 0{,}158} \cdot \sin 90{,}00°} = 0{,}00028694 \text{ m}^2 = 2{,}87 \text{ cm}^2$$

The minimum area of shear reinforcement: EN-1992-1-1 9.4.3 (2) (9.11.)

$$A_{sw,min} = \frac{0{,}08 \cdot \frac{\sqrt{f_{ck}}}{f_{ywk}} \cdot s_r \cdot u_1}{1{,}5 \cdot \sin\alpha + \cos\alpha} = \frac{0{,}08 \cdot \frac{\sqrt{16}}{500} \cdot 0{,}1185 \cdot 3{,}3027}{1{,}5 \cdot \sin 90{,}00° + \cos 90{,}00°} = 0{,}00016699 \text{ m}^2 = 1{,}67 \text{ cm}^2$$

where f_{ck} and f_{ywk} is measured in N / mm^2

k_{max} = 1,5 EN-1992-1-1/A1:2014 6.4.5 (1)

$$A_{sw} = \max [A_{sw,c} \; ; \; A_{sw,min}] = \max [2{,}87 \; ; \; 1{,}67] = 2{,}87 \text{ cm}^2$$

$$v_{Rd,cs} = \text{Min} \left(0{,}75 \cdot v_{Rd,c} + 1{,}5 \cdot \frac{d}{s_r} \cdot A_{sw} \cdot f_{ywd,eff} \cdot \frac{1}{u_1 \cdot d} \cdot \sin\alpha = \right.$$

$$= 0{,}75 \cdot 494{,}55 + 1{,}5 \cdot \frac{0{,}158}{0{,}1185} \cdot 0{,}00028694 \cdot 289500 \cdot \frac{1}{3{,}3027 \cdot 0{,}158} \cdot \sin 90{,}00° = 689{,}29 \; ; \; k_{max} \cdot v_{Rd,c} = 1{,}5 \cdot 494{,}55 = 741{,}82 \Big) =$$

$$= 689{,}29 \text{ KPa}$$

$$v_{Rd,cs} = 689{,}29 \text{ KPa} \ge v_{Ed} = \beta \cdot \frac{V_{Ed}}{u_1 \cdot d} = 1{,}350 \cdot \frac{266{,}43}{3{,}3027 \cdot 0{,}158} = 689{,}28 \text{ KPa}$$

passed

The control perimeter at which no shear reinforcement is required: EN-1992-1-1 6.4.5. (4)

$$u_{out} = \frac{\beta \cdot v_{Ed}}{v_{Rd,c} \cdot d} = \frac{1{,}350 \cdot 689{,}28}{494{,}55 \cdot 0{,}158} = 4{,}6032 \text{ m} \quad \text{EN-1992-1-1 (6.54)}$$

The distance between the circumference of the loaded area and the u_{out} perimeter:

a_{out} = 0,54032 m

Number of link leg perimeters (reinforcement circles):

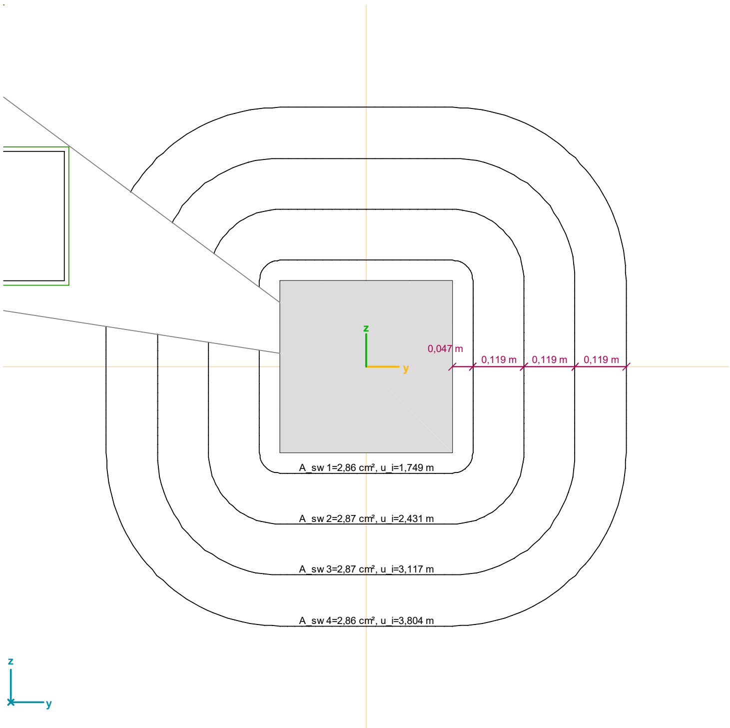
$$n_{sr} = \frac{a_{out} - k_{out} \cdot d - s_0}{s_r} + 1 = \frac{0{,}54032 - 1{,}5 \cdot 0{,}158 - 4{,}7}{0{,}1185} + 1 = 3{,}163 \ge n_{sr,min} = \frac{1{,}5 \cdot d}{s_r} = \frac{1{,}5 \cdot 0{,}158}{0{,}1185} = 2 \rightarrow n_{sr} = 4$$

$A_{sw,1} = A_{sw}$ = 2,87 cm²

$A_{sw,2} = A_{sw}$ = 2,87 cm²

$A_{sw,3} = A_{sw}$ = 2,87 cm²

$A_{sw,4} = A_{sw}$ = 2,87 cm²


0,047 m
0,119 m
0,119 m
0,119 m
A_sw 1=2,86 cm², u_i=1,749 m
A_sw 2=2,87 cm², u_i=2,431 m
A_sw 3=2,87 cm², u_i=3,117 m
A_sw 4=2,86 cm², u_i=3,804 m
z
y